AXIAL DISPERSION IN TUBULAR REACTOR

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Received January 22nd, 1974

The tubular plug flow reactor is suitable for many types of reactions. But in the actual design of reactors, in which the flow would be as closely to plug flow as possible many problems are encountered. A satisfactory reactor is a horizontal pipe which is only suitable for fast reactions as it is not possible to increase its length without limitation. By bending the pipe into the shape of a helix much more advantageous spacial arrangement is obtained for which the axial dispersion in comparison with the straight pipe is small. But its disadvantage is the difficult — if not impossible — cleaning of the solid deposits formed on the inner walls. For these reasons has been the reactor formed by a combination of straight parts and 180° bends in the "helical" arrangement, which we expect should combine the advantages of straight pipes-small dispersion with the easier cleaning.

In this study the axial dispersion is determined experimentally in the described reactor both in the transition and turbulent regions. The obtained results should be applied to its optimum geometrical arrangement.

In flow of fluids through circular pipes bent into the form of a helix, a secondary flow appears due to the action of the centrifugal force (it can be characterized as a circulating flow in the plane perpendicular to the pipe axis) which is combined with the velocity distribution in the axial direction so that fluid particles follow a spiral path in the pipe axis.

For laminar flow, when the secondary flow is combined with a parabolic velocity distribution, Dean[1] has obtained analytical expressions of the components of this spiral flow by solving the Navier–Stokes equations analytically for incompressible viscous fluid. By solving these equations under certain simplifying assumptions Ruthven[2] succeeded fully to calculate the mean residence time of fluid particles or, by combination with the equation for dispersion mass transfer, determined[3] relations for calculation of the dispersion coefficient D.

In the turbulent region a similar calculation is not yet available since the solution of Navier–Stokes equations for bent pipes has not been succesfull under the given flow conditions. We are thus limited only to experimental data for determination of axial dispersion. Qualitatively, it is possible to say: At turbulent fluid flow in circular pipes bent into the shape of a helix the axial dispersion is affected by two counteracting mechanisms. The first promotes axial dispersion due to the velocity profile and due to various lengths of particle paths in bent pipes. The second mechanism which is the result of secondary flow impedes axial dispersion due to transverse mixing. In the same way acts the radial dispersion resulting from turbulent diffusivity.

It is obvious that the flow character in the combined reactor (formed by straight pipes and bends) is complicated. The velocity profile and the secondary flow are in bends with different angles of the bend different while a great role is played by various lengths of straight parts between bends.

Many authors have paid attention to correlations of the dispersion number D/ud (where D is the dispersion coefficient and u is the mean flow velocity and d the pipe diameter) both for straight pipes and reactors with bent parts. Conclusions of some lately made studies, which concern dispersion of liquid characterized by the dispersion number D/ud in the transition and turbulent regions of straight pipes, are given further. These are first of all studies by Kenney and Thwaites[4], Woodhead, White and Yesberg[5] and Sittel and coworkers[6]. First two papers are presenting the results in the form of graphs (Fig. 1) while Sittel and coworkers[6] present for D the relation

$$D = 3{\cdot}594 \,.\, 10^{-6} \,.\, \mathrm{Re}^{0{,}764} \, (\mathrm{m^2\, s^{-1}}) \,, \qquad (1)$$

for $\mathrm{Re} > 4 \,.\, 10^4$.

As one of the fundamental studies can be considered the paper by Koutsky and Adler[7] in which is studied the axial dispersion in tubular reactors bent into the shape of a helix. The most important conclusion they have made is that dispersion in the helix is in the turbulent region also lower than in the straight pipe due to the fully developed secondary flow. In this study are also given graphs from which the dispersion number can be determined for the given Re number, ellipticity (*i.e.* ratio of lengths of both axes of the ellipse into which the pipe is at its bending flattened) of the pipe forming the helix and the radius of curvature. Aunický[8], Cassel and Perona[9] and Park and Gomezplata[10] have been measuring the dispersion of bends 45—90° and pipe systems with such bends. They all claim that the presence of 90° bends in the piping increases the axial dispersion. Although it is not possible to compare directly the flow in the helix and in the bends alone, completely contradictory conclusions on axial dispersion in both types of arrangements are surprising. In our last study[11] the tubular reactor has been modelled with a hose bent around two cylindres. The distance of both cylindres has been altered from the finite distance to zero *i.e.* the axis of both cylinders have merged and the hose has bent on one cylinder only in a form of a helix. The results demonstrate that the axial dispersion of this system decreased with the increase of the number of 180° bends but was always, *i.e.* also for a helix, greater than for the straight pipe (Fig. 1). The dispersion number was calculated for parameters of this helical reactor (length 58 m, pipe diameter 25 mm, elipticity = 1, diameter of the cylinder 32 cm) according to the graph given in the study[7]. The calculated values were lower than the experimental results, but nevertheles greater than for the straight pipe. For a reliable design of a considered combined reactor type it proves to be necessary to make more detailed measurements of axial dispersion on a model unit at different geometrical arrangements.

EXPERIMENTAL

Experimental apparatus. The tubular reactor was modelled by a polyethylene hose with inside diameter 8 mm 26·32 m long which was helically bent around a pair of steal cylinders. Three models in total have been measured with diameters of cylinders 60, 108 and 150 mm, *i.e.* the radius of curvature of bent parts has differred while the diameter of the reactor pipe remained the same. The arrangements have made possible to change the distance of both cylinders and thus the ratio of lengths of straight and bent parts of the reactor. The maximum distance of axis of cylinders was 2·20 m, the second limiting case has been represented by a reactor formed only by a helix. The two-way valve situated at the inlet into the tubular pipe reactor has made possible an instantaneous feeding of the impuls of the tracer solution beside the flow of the liquid simulating the reaction medium. For liquid has been used the deionised water. The impuls of the tracer (ostazine brillant red S-5B) was realized by switching the two-way valve from the pressure vessel with water to the pressure vessel with the tracer solution with which was simultaneously connected the circuit of the time switch and of the regulator which at the chosen time interval (several tenths of a second) has returned the valve into the original position. The outlet from the reactor has been analyzed continuously colorimetrically by the modified collorimeter Spekol.

Experimental measuring procedure. For the selected geometrical arrangement of the reactor and after steadying the water flow rate the impuls of the tracer solution was introduced and the *C*-curve was registered on the recording of the colorimeter. By the method of moments[12] were calculated values of the dispersion number. Each experiment has been performed three times and for the following calculations the arithmetic mean of these three measurements has been taken. The velocity of flow inside the reactor has been altered so that six values of Re number in the range from 5000 to 20000 have been obtained. Calibration of the colorimeter and all measurements have been made at the wave length 520 nm.

We have assumed at the evaluation of experimental data that dispersion at the inlet into the reactor is negligible in comparison with the dispersion at the outlet which was reliably ensured by the type of the two-way cock used. Also the phenomena appearing at the sudden stoppage of liquid delivery were also neglected. An uniform distribution of bends as possible along the reactor length has been arranged for. With regard to the small value of the dispersion number,

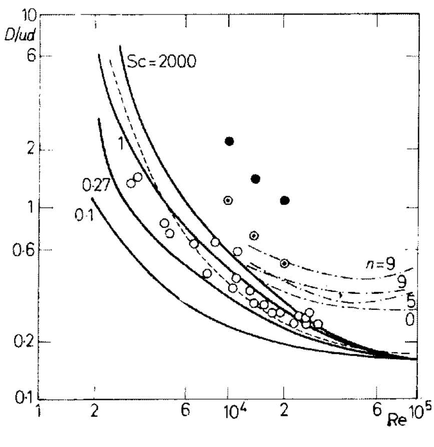

FIG. 1

Experimental Axial Dispersions of Straight Pipes and Pipes with Bends

——— Theoretical relations for straight pipes[5] (Sc = Schmidt number) – – – theoretical relations for straight pipes[4]; –·–·– experimental data for pipes with 90° bends[9] for various number of bends *n*; ○ experimental data[4], ● experimental data for a helix[11]; ⊙ calculated according to[7] for experimental data of a helix[11].

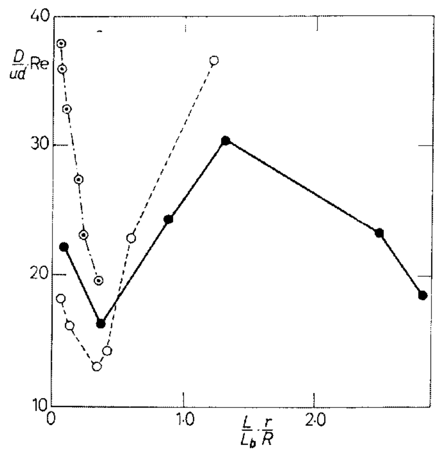

FIG. 2

Dependence of the Dispersion Number on Geometrical Parameters of the Reactor

R, mm: ● 36; ○ 60; ⊙ 81.

for calculation was used the simplified relation

$$D/ud = \delta^2/2(L/d)\,, \tag{2}$$

where D is the dispersion coefficient, u mean velocity of fluid in the pipe, d diameter of reactor, L length of reactor, and δ^2 dimensionless variance of values of the C-curve. Validity of the relation (2) for our conditions has been proved by calculation. All the arrangements used are summarized in Table I.

RESULTS AND DISCUSSION

The dispersion number D/ud is for all used arrangements of the reactor with the bends 180° greater than for a straight pipe and for the given experimental conditions its values are in the range from 0·79 to 5·62. With increasing Re number the value of the dispersion number decreases in the range of considered experimental values.

TABLE I

Experimental Conditions (D/ud) Re

Radius of curvatures mm	Distance of axes of cylinders m	Total length of bends L_b, m	$(Lr)/(L_bR)$	$[D/(ud)]$ Re
36	2·16	1·03	2·836	18 530
36	1·38	1·17	2·497	23 260
36	0·96	2·28	1·281	30 210
36	0·63	3·35	0·861	24 340
36	0·16	7·61	0·384	16 110
36	0	25·68	0·114	22 230
60	2·20	1·44	1·217	36 600
60	1·33	2·90	0·604	22 960
60	0·98	4·18	0·419	14 210
60	0·66	4·99	0·351	13 210
60	0·18	13·30	0·132	16 200
60	0	25·52	0·0686	18 160
81	2·20	3·45	0·377	19 190
81	1·38	5·06	0·257	23 100
81	1·00	6·11	0·213	27 400
81	0·72	12·50	0·104	32 700
81	0·24	18·00	0·0722	35 900
81	0	25·70	0·0506	37 600

The graphical dependence of the dispersion number on Re number for individual arrangements resulted in the correlation relation

$$D/ud = C\,\mathrm{Re}^{m}\,, \tag{3}$$

where C and m are constants.

Values of the correlation coefficients of the given relation were about equal to 0·98 with the constants C and m determined by the least square method. As it was not possible to prove the dependence of the constant m on any other quantity it has been concluded that differences in its values are the result of experimental errors. Its average value is $-1{\cdot}003$. In the following we are considering this quantity as equal to $-1{\cdot}0$ which is within the 95% interval of reliability. Thus it holds

$$(D/ud)\,\mathrm{Re} = C\,. \tag{4}$$

Coefficient C is characterizing the geometrical arrangement of the reactor and is for individual arrangements only a function of this geometry.

For determination of the effect of geometrical arrangement on coefficient of axial dispersion, the left hand side quantity of Eq. (*4*) has been plotted in dependence on a number of dimensionless quantities representing the geometrical similarity. As the most suitable has proved to be the number including the length quantities $(Lr)/L_bR$ (where L_b is the total length of bent parts of the pipe, L length of the reactor pipe, r internal diameter of the reactor pipe and R radius of curvature of bends) (Fig. 2).

We did not succeed in expressing the curves analytically. But it is obvious that in the range of the considered geometrical arrangements of the reactor there exists a narrow region where the product of dimensionless numbers (D/ud) Re is passing through the minimum. As resulted from the study[7] this minimum does not correspond to the geometry of the helix but to the value $(Lr)/(L_bR) = 0{\cdot}3 - 0{\cdot}5$. This phenomenon is quantitatively explained in the following manner: Let us consider the tubular reactor with one diameter of cylinders and with the variable length of straight parts in between the 180° bends and let us take into consideration the mechanism affecting the axial dispersion. If the cylinders are situated sufficiently far one from the other the axial dispersion is relatively small and is approaching that of the straight pipe. This is because the number of bends is small and the length of straight parts is sufficient for the flow to become steady behind the bend with the velocity profile fully developed. If the cylinders are moved closer together, axial dispersion slightly increases due to the increasing number of bends as the secondary flow is still insignificant with the mechanism supporting dispersion in control.

From some value of the ratio of the total length to the length of bends, dispersion begins to decrease as the effect of secondary flow acts more strongly (length of straight parts behind the bends is not sufficient for steadying the helical flow originated in the bends and thus for development of a perfect velocity profile). The mechanism suppressing dispersion prevails up to a certain value of the ratio $(Lr)/(L_b R)$ at which the dispersion is reaching its minimum. With the cylinders pushed still closer together (when the arrangements become close to that of a helix) dispersion again increases as the effect of secondary flow begins to be overlapped by a considerably different path lengths of particles on the internal and external circumferences. Together with the different ratio of lengths of straight and curved parts acts the effect of different radius of curvature. This takes place with individual maxima and minima appearing at various geometrical arrangements while the product of dimensionless numbers (L/L_b) and (r/R) is still about constant.

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Translated by M. Rylek.